Adaptive Nonlinear Control of Single-Phase Inverters for UPS Applications: Average Performance Analysis

A. Abouloifa, I. Lachkar, F. Giri, A. Hamdoun

Abstract— This paper focuses on the problem of controlling DC-AC switched power converters used in Uninterruptible Power Supply (UPS). The control objective is to generate, at the system output, a sinusoidal voltage with amplitude and frequency fixed by the reference signal. The controller is designed using the backstepping approach in its adaptive versions and the Lyapunov stability argument. Unlike the existing formulations, the approach also affords additional analysis based on averaging theory. Adaptive versions is designed and shown to yield quite interesting tracking and robustness performances with respect to load changes as it is illustrated by simulation result in Matlab/Simulink.

I. INTRODUCTION

In recent years, there has been a considerable increase in the use of uninterruptible power supply (UPS) to provide a low output voltage distortion, fast dynamic response, high reliability and continuous power supply systems especially for sensitive and critical loads, which cannot afford have unexpected power failure. Critical loads such as computer systems, hospitals and air line reservation systems need UPS [1]. Low total harmonic distortion (THD) and high efficiency are commonly required in high power applications, such as three-phase inverter systems. Nonlinear loads and non-sinusoidal currents can cause more voltage drops on the supply network impedance resulting in unbalanced conditions. They also cause electromagnetic Interference (EMI) and resonances. The harmonics have negative influence on the control and automatic equipment protection systems and other electrical loads, resulting in reduced reliability and availability [2].

Any UPS system has two operating modes: bypass mode and backup mode. Ideally, a UPS should be able to deliver a regulated sinusoidal output voltage with low total harmonic distortion (THD) during the two modes, even when feeding nonlinear loads [3, 4]. Several controllers including repetitive control [5, 6], hysterics regulation [7, 8], predictive control [4] and multi-loop feedback control [11], have been proposed to obtain an output voltage with low distortion for single and three-phase UPS inverter.

Many papers have been so far published in the field of control and topologies. Deadbeat control on both output voltage and inductor or capacitor current for three-phase PWM inverter has been proposed, either in multi-loop configurations or in a conventional linear state feedback. Performance of two-stage cascade output filter for single-phase voltage source UPS inverter has been investigated in and compared with an LC output filter. In [5] a unified control scheme as well as a novel connection arrangement is developed to simplify the inverter circuit for design of line interactive UPS without load current sensors. A digital control scheme of three-phase UPS inverter based on multi-loop control strategy consisting of a filter capacitor current and output voltage is proposed in [16]. The technique also includes a load predictive feed-forward loop in a voltage controller and an output voltage feed forward loop in a current controller. In [17], linear and nonlinear adaptive control strategies for three-phase UPS inverters have been presented. An on-line adaptive learning algorithm has been also described which promotes steady state controller stability.

The remainder of this paper can be outlined as follows. In section 2, a mathematical model of the proposed system including three-phase PWM inverter, nonlinear load, control system and multiple-filter is described. In section III, a small signal equivalent circuit and transfer function dq model of the system are derived. Simulation results are reported and discussed in section IV. Finally, a brief summary is given in section V.
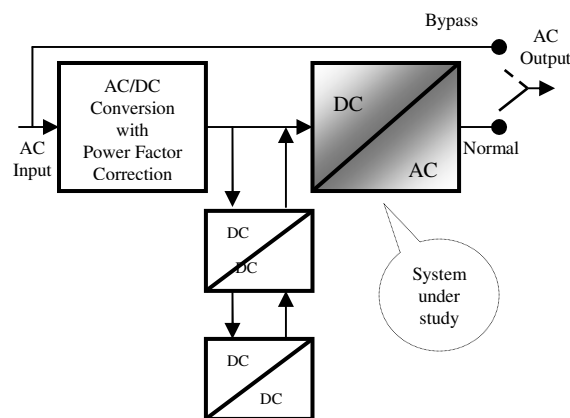

Fig. 1. Uninterruptible power Supply
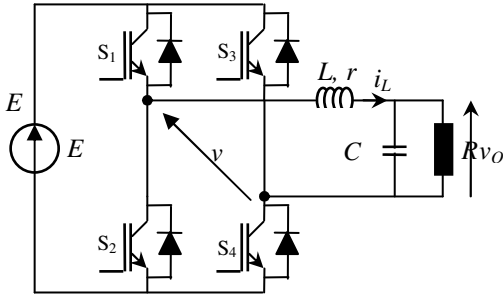
II. DYNAMIC MODEL OF PWM INVERTER

Fig.2. single phase inverter

A typical single-phase inverter under study has the structure shown in figure 2. It consists of a single-phase full bridge where the switches can be IGBT or MOSFET. From the AC side, the converter is connected, in parallel with a load to be supplied through a filtering inductor-capacitor L-C. The switch-based inverter operates in accordance to the well known Pulse Width Modulation principle [10]. The duty cycle of the inverter varies between ± 100% and the amplitude of the inverter voltage V is proportional with the dc-link voltage E and the duty cycle. A bipolar PWM voltage modulation type is used because this method offers the advantage of effectively minimising the switching frequency of the inverter in comparison with the unipolar PWM technique.

Applying the usual Kirchhoff's laws to the single-phase shunt APF one gets:

$$C\frac{dv_o}{dt} = i_L - \frac{v_o}{R} \tag{1a}$$

$$L\frac{di_L}{dt} + ri_L = v - v_o \tag{1b}$$

The inverter undergoes the equations:

$$v = \mu E \tag{2}$$

where the switching function $\mu \in \{-1,1\}$ of the inverter is defined by:

$$\mu = \begin{cases} 1 & if \ \ (s_1,s_4) \ is \ ON \ \ ; \ \ (s_2,s_3) \ is \ OFF \\ -1 & if \ \ (s_1,s_4) \ is \ OFF \ \ ; \ \ (s_2,s_3) \ is \ ON \end{cases} \tag{3}$$

Combining (1) and (2), one obtains the instantaneous model of the converter:

$$\frac{dv_o}{dt} = \frac{1}{C}i_L - \frac{1}{RC}v_o \tag{4a}$$

$$\frac{di_L}{dt} = \frac{-1}{L}v_o - \frac{r}{L}i_L + \frac{\mu}{L}E \tag{4b}$$

The model (4a-b) is useful for building up an accurate simulator of the inverter. However, it cannot be based upon in the control design as it involves a binary control input, namely μ. This kind of difficulty is generally coped with by resorting to average models. Signal averaging is performed over cutting intervals. The obtained average model is the following:

$$\dot{x}_1 = -\frac{1}{RC}x_1 + \frac{1}{C}x_2 \tag{5a}$$

$$\dot{x}_2 = -\frac{1}{L}x_1 - \frac{r}{L}x_2 + \frac{E}{L}u \tag{5b}$$

where x_1, x_2, and u denote the average values, over cutting periods, of the signals v_o, i_L and μ. In the system (5a-b) the mean value u of μ turns out to be the system control input.

III. BACKSTEPPING CONTROL OF FULL BRIDGE

The backstepping approach is a recursive design methodology [12]. It involves a systematic construction of both feedback control laws and associated Lyapunov functions. The controller design is completed in a number of steps, which is never higher than the system order.

The load resistance R in the model (5a-b) may be subject to step changes. The controller to be designed should involve an on-line estimation of the unknown parameter:

$$\theta = \frac{1}{RC} \tag{6}$$

The corresponding estimate is denoted $\hat{\theta}$, and the parameter estimation error is:

$$\tilde{\theta} = \theta - \hat{\theta} \tag{7}$$

The aim is to directly enforce the (average) capacitor voltage output x_1 to track a given reference voltage $x_1^*(t) = V \sin(\omega t)$ despite the system parameter uncertainties.

Following the backstepping technique, a controller is designed in two steps because the controlled system has the relative degree equal two with respect to the output signal

x_1. The controlled have to be developed based on the following averaging model:

$$\dot{x}_1 = -\theta x_1 + \frac{1}{C} x_2 \tag{8a}$$

$$\dot{x}_2 = -\frac{1}{L} x_1 - \frac{r}{L} x_2 + \frac{E}{L} u \tag{8b}$$

Step 1: stabilization of the system (8a).

Let us introduce the tracking error on the output voltage:

$$z_1 = x_1 - x_1^* \tag{9}$$

where x_1^* denotes the corresponding reference signal. The later is bounded and as smooth as necessary. Using (8a), time derivation of z_1 yields the following error dynamics:

$$\dot{z}_1 = w_1\left(\tilde{\theta} + \hat{\theta}\right) + \frac{1}{C} x_2 - \frac{dx_1^*}{dt} \tag{10}$$

where $w_1 = -x_1$ is the first regressor constant.

In the above equation, the quantity $\left(x_2/C\right)$ stands as a virtual control variable. Then, z_1 can be regulated to zero if $\left(x_2/C\right) = \sigma$ where σ is an appropriate stabilizing function to be defined. To this end, consider the following Lyapunov candidate function:

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2\gamma} \tilde{\theta}^2 \tag{11}$$

where $\gamma > 0$ is any real constant, called parameter adaptation gain.

Using (10), time derivative of (11) is obtained, using the fact that $\dot{\tilde{\theta}} = -\dot{\hat{\theta}}$, as follows:

$$\dot{V}_1 = \left(\frac{x_2}{C} + w_1\hat{\theta} - \frac{dx_1^*}{dt}\right) z_1 - \left(\frac{\dot{\hat{\theta}}}{\gamma} - w_1 z_1\right) \tilde{\theta} \tag{12}$$

This shows that, for the $\left(z_1, \tilde{\theta}\right)$-system to be globally asymptotically stable, it is sufficient to choose the (virtual) control signal and the parameter adaptation law so that $\dot{V}_1$ to be negative definite. In view of (12) this suggests the following choices:

$$\frac{x_2}{C} + w_1\hat{\theta} - \frac{dx_1^*}{dt} = -c_1 z_1 \tag{13a}$$

$$\dot{\hat{\theta}} = \gamma \tau_1 \tag{13b}$$

where c_1 is a design parameter, and $\tau_1 = w_1 z_1$ is the first tuning function. Equation in (13a) implies that $\left(x_2/C\right) = \sigma$ where σ is the first stabilizing function:

$$\sigma = -c_1 z_1 - w_1\hat{\theta} + \frac{dx_1^*}{dt} \tag{14}$$

As $\left(x_2/C\right)$ in not the actual control input, a new error variable z_2 between the virtual control and its desired value σ (stabilizing function) is introduced:

$$z_2 = \left(x_2/C\right) - \sigma \tag{15}$$

Then, equation (10) becomes, using (14) and (15):

$$\dot{z}_1 = -c_1 z_1 + z_2 + w_1\tilde{\theta} \tag{16}$$

In the same way

$$\dot{V}_1 = -c_1 z_1^2 + z_1 z_2 - \left(\frac{\dot{\hat{\theta}}}{\gamma} - w_1 z_1\right) \tilde{\theta} \tag{17}$$

Step 2: stabilization of the sub-system $\left(z_1, z_2, \tilde{\theta}\right)$.

The objective now is to enforce the error variables $\left(z_1, z_2\right)$ to vanish. To this end, let us first determine the dynamics of z_2. Deriving (15) and using (8b), (14) and (16), one obtains:

$$\dot{z}_2 = w_2\tilde{\theta} + w_1\dot{\hat{\theta}} + \frac{E}{LC} u + \psi \tag{18}$$

where $w_2 = c_1 w_1$ is the second regressor constant and ψ is defined as follows :

$$\psi = -\frac{1}{LC} x_1 - \frac{r}{LC} x_2 - c_1^2 z_1 + c_1 z_2 - \frac{d^2 x_1^*}{dt^2} \tag{19}$$

We are finally in a position to make a convenient choice of the parameter update law and feedback control to stabilize the whole system with state vector $\left(z_1, z_2, \tilde{\theta}\right)$. Consider the augmented Lyapunov function candidate.

$$V_2 = V_1 + \frac{1}{2} z_2^2 \tag{20}$$

Time derivative of V_2 is given as follows

$$\dot{V}_2 = -c_1 z_1^2 + z_2\left(z_1 + w_1\dot{\hat{\theta}} + \frac{E}{LC} u + \psi\right) - \tilde{\theta}\left(\frac{\dot{\hat{\theta}}}{\gamma} - \tau_2\right) \tag{21}$$

where $\tau_2 = \tau_1 + w_2 z_2$ is the second tuning function.

As our goal is to make $\dot{V}_2$ non-positive definite, this suggests choosing u so that the bracketed term, on the right side of (21), is equal to $-c_2 z_2$ and the adaptation law $\dot{\hat{\theta}} = \gamma \tau_2$. c_2 is a positive constant of synthesis.

$$\dot{\hat{\theta}} = \gamma \tau_2 \tag{22}$$

Doing so, one gets, using (21), the following control law:

$$u = -\frac{LC}{E}\left(z_1 + c_2 z_2 + w_1 \gamma \tau_2 + \psi\right) \tag{23}$$

Indeed, with this choice, the dynamics of Lyapunov function is reduced to:

$$\dot{V}_2 = -c_1 z_1^2 - c_2 z_2^2 \tag{24}$$

Proposition 1. Consider the control system consisting of the system (5a-b) and the control law (23). The inner closed-loop system undergoes, in the $(z_1, z_2, \tilde{\theta})$-coordinates, the following equation:

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{\tilde{\theta}} \end{pmatrix} = \begin{pmatrix} -c_1 & 1 & w_1 \\ -1 & -c_2 & w_2 \\ -\gamma w_1 & -\gamma w_2 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ \tilde{\theta} \end{pmatrix} \tag{25}$$

Considering $w_1 = -x_1$, $w_2 = c_1 w_1$ and $x_1^* = V \sin(\omega t)$, then, in the $Z = (z_1, z_2, \tilde{\theta})^T$-coordinates the dynamics of the closed loop system is expressed as follows:

$$\dot{Z}(t) = f(Z, t) \tag{26}$$

where

$$f(Z,t) = \begin{pmatrix} -c_1 z_1 + z_2 - z_1 \tilde{\theta} - \tilde{\theta} V \sin(\omega t) \\ -z_1 - c_2 z_2 - c_1 z_1 \tilde{\theta} - c_1 \tilde{\theta} V \sin(\omega t) \\ \gamma(V \sin(\omega t) + z_1)(z_1 + c_1 z_2) \end{pmatrix}$$

Notice that the above system has an equilibrium at $(z_1, z_2, \tilde{\theta}) = (0, 0, \tilde{\theta}_0)$ where $\tilde{\theta}_0 \in \mathbb{R}$. The stability properties of this equilibrium are analyzed in the following theorem:

Theorem 1: Consider the closed-loop system consisting of a half bridge DC/AC converter system represented by the averaging model (8a-b) subject to uncertain load resistor R, and the controller composed of the adaptive control law (23) the parameter update law (22) and desired trajectory $x_1^*(t) = V \sin(\omega t)$ of the output voltage x_1. Then, one has:
(i) the closed-loop system is globally asymptotically stable;
(ii) the estimation error $\tilde{\theta} = \theta - \hat{\theta}$ converges (in averaging)

to finite value $\tilde{\theta}_0$ which means that the tracking error $z_1 = x_1 - x_1^*$ converges (in averaging) to zero, this propriety ensures tight regulation under uncertainties.

Introduce the notation $\varepsilon = 1/\omega_s$. Then, there exists a positive real ε^* such that if $\varepsilon \le \varepsilon^*$, the tracking errors (z_1, z_2) and the estimation error $\tilde{\theta}$ are harmonic signals continuously depending on ε, i.e. $z_1 = z_1(t, \varepsilon)$, $z_2 = z_2(t, \varepsilon)$, $\tilde{\theta} = \tilde{\theta}(t, \varepsilon)$. Furthermore, when $\varepsilon \to 0$, all errors converge. Specifically:

$$\lim_{\varepsilon \to 0} z_1(t, \varepsilon) = 0, \ \lim_{\varepsilon \to 0} z_2(t, \varepsilon) = 0, \ \lim_{\varepsilon \to 0} \tilde{\theta}(t, \varepsilon) = \tilde{\theta}_0.$$

Proof. The result of Proposition 1 does hold implying that the errors $(z_1, z_2, \tilde{\theta})$ undergo equation (26). The stability of the time-varying system (26) will now be analyzed making use of the averaging theory (e.g. Khalil, 2003). To this end, introduce the time-scale change $\tau = \omega_s t$. It is readily seen from (26) that $Z_0(\tau) \overset{def}{=} Z(\tau/\omega_s)$ undergoes the differential equation:

$$\dot{Z}_o(\tau) = \varepsilon f_0(Z_o, \tau) \tag{27}$$

with:

$$f^0(Z^0, t) = \begin{pmatrix} -c_1 z_1^0 + z_2^0 - z_1^0 \tilde{\theta}^0 - \tilde{\theta}^0 V \sin(\tau) \\ -z_1^0 - c_2 z_2^0 - c_1 z_1^0 \tilde{\theta}^0 - c_1 \tilde{\theta}^0 V \sin(\tau) \\ \gamma(V \sin(\tau) + z_1^0)(z_1^0 + c_1 z_2^0) \end{pmatrix}$$

Clearly, the above time-varying matrices are periodic with period 2π. Now, let us introduce the average functions:

$$\overline{Z}^0 = \lim_{\varepsilon \to 0} \frac{1}{2\pi} \int_0^{2\pi} Z^0(\tau) d\tau$$

It follows from (27) that:

$$\dot{\overline{Z}}^0(\tau) = \varepsilon f\left(\overline{Z}^0\right) \tag{28}$$

with:

$$\overline{f}^0\left(\overline{Z}^0\right) = \begin{pmatrix} -c_1 \overline{z}_1^0 + \overline{z}_2^0 - \overline{z}_1^0 \overline{\tilde{\theta}}^0 \\ -\overline{z}_1^0 - c_2 \overline{z}_2^0 - c_1 \overline{z}_1^0 \overline{\tilde{\theta}}^0 \\ \gamma \overline{z}_1^0 \left(\overline{z}_1^0 + c_1 \overline{z}_2^0\right) \end{pmatrix}$$

In order to get stability results regarding the system of interest (26), it is sufficient (thanks to averaging theory) to analyze the following averaged system (28). To this end, notice that (28) has a unique equilibrium at:

$$\bar{Z}^0 = \begin{pmatrix} 0 \\ 0 \\ \tilde{\theta}_0 \end{pmatrix}$$

IV. SIMULATION RESULTS

The controlled system is a half bridge inverter with the parameters listed in Table 1. The experimental bench is simulated by using MATLAB software. In this respect, all power components are simulated using the relevant Simulink/SimPowerSystems. Then, the capability of the proposed controller will be illustrated through the different simulation tests.

Table 1. Inverter characteristics

Components	Symbol	Value
Input voltage	E	$400V$
Inductor	L, r	$5mH$, $10m\Omega$
Capacitor	C	$10\mu F$
Load resistor	R	$25\Omega...50\Omega$

The proposed adaptive control design is considered with the following numerical values of design parameters which proved to be suitable:

Table 2. Controller parameters

| Symbol	Value
c_1	4×10^4
c_2	1×10^4
γ	2×10^{-8}

These have been selected using the common try-error method that consists in increasing the parameter values until a satisfactory compromise, between rapidity of responses and control activity, is achieved. The behavior of such a system is illustrated in figures 3 to7. These figures illustrate the behavior of controlled system with an output voltage reference $x_1^*(t) = V\,sin(\omega t)$ ($V=220\sqrt{2}$, $\omega=100\pi$ rad.s^{-1}) and successive load step changes, the resistance can change between 25Ω and 50 Ω, yielding variation of 50% of the power of the AC bus. As it can be seen, despite the load resistor uncertainty, the controller behavior is satisfactory. Fig. 3 shows a tight voltage regulation under step load changes. Fig. 4 depicts the parameter θ changes and its estimated value $\tilde{\theta}$. This figure illustrates a perfect estimation of uncertain parameter, with negligible steady state error and fast transient response. Furthermore, it is observed that the estimated parameter oscillate at the frequency $2\omega_s$. Figures 5, 6 and 7 show, respectively, inductor current i_L, input current i_E and the control law u.
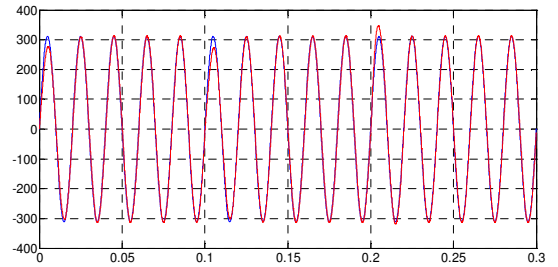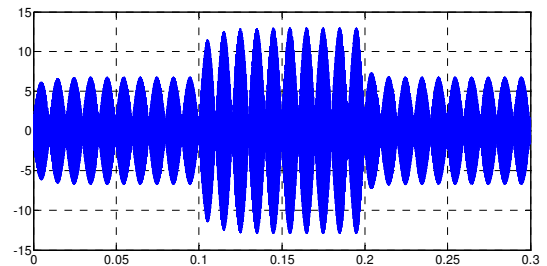

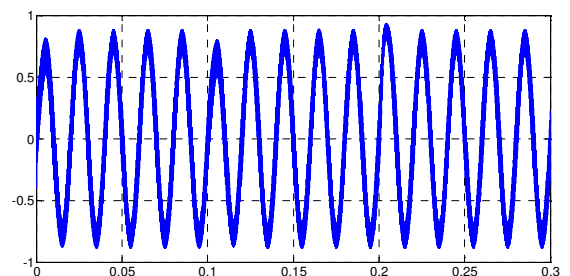
Fig. 3. Output voltage v_o and its reference x_1^*
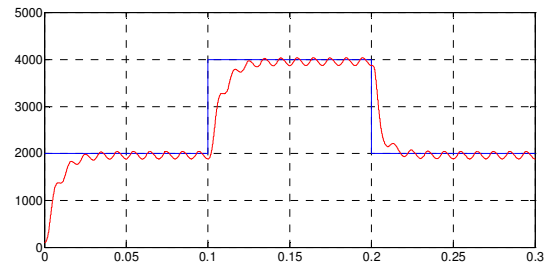
Fig. 4. Uncertain parameter and its estimate
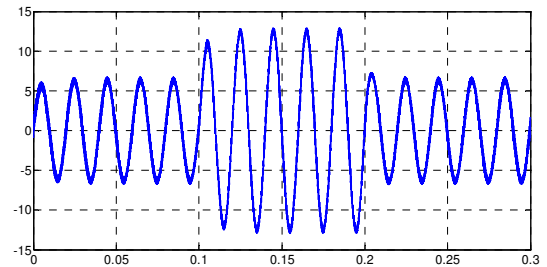
Fig. 5. Inductor current i_L

Fig. 6. input current i_E

Fig. 7. Control law

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